

The Role of the QCD Vacuum in the Heavy-Quark Bound State Dynamics^a

NORA BRAMBILLA and ANTONIO VAIRO

*Institut für Theoretische Physik, Universität Heidelberg
Philosophenweg 16, D-69120 Heidelberg, Germany
and*

*Theory Division CERN, 1211 Geneva 23, Switzerland
E-mail: nora.brambilla@cern.ch, antonio.vairo@cern.ch*

The effective field theory approach allows a rigorous disentangling of high and low energy effects in the heavy quarkonium dynamics. Focusing in particular on the spectrum, we describe the nature of the non-perturbative effects and discuss our present knowledge of them.

1 Introduction

It is well known that the complex structure of the QCD vacuum can, in some regimes, be parametrized by local vacuum condensates, i.e. the expectation values of operators where all the perturbative vacuum fluctuations are taken out at the best of our ability¹. The first attempt to calculate the effect of non-perturbative vacuum condensates on the energy levels of heavy quarkonia was performed in². The leading contribution (in α_s and Λ_{QCD} , the scale of non-perturbative physics) of the vacuum condensates to Coulombic n, l heavy quarkonium states reads²:

$$\delta E_{nl}^{\text{V-L}} = m \frac{\epsilon_n n^6 \pi^2 G_2}{(m C_F \alpha_s)^4}, \quad (1)$$

where ϵ_n is a (known) number of order 1 and $G_2 \equiv \langle (\alpha_s/\pi) F_{\mu\nu}^a(0) F^{a\mu\nu}(0) \rangle$ is the gluon condensate. The above expression for the corrections to Coulombic energy levels displays two relevant characteristics: 1) it is a correction of non-potential type, like the Lamb shift in QED; 2) it grows like $\sim n^6$, thus being out of control for levels beyond the ground state. It was soon realized^{3,4} that the strong growth in n could be corrected to some extent by considering non-local gluon condensates

$$G_2(x) \sim \left\langle \frac{\alpha_s}{\pi} F_{\mu\nu}^a(x) \phi(x, 0)_{ab}^{\text{adj}} F^{b\mu\nu}(0) \right\rangle, \quad (2)$$

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where $\phi(x, 0)^{\text{adj}}$ is the Schwinger line (in the adjoint representation) connecting x with 0. These were understood as due to the presence of a fluctuating gluonic background with a characteristic time length $T_g \sim \Lambda_{\text{QCD}}^{-1}$. It is apparent that the local condensates stem from an expansion of the non-local ones in cases in which the correlation length is large with respect to the other physical scales of the system.

Heavy quarkonium is a non-relativistic bound system. Besides Λ_{QCD} , it is characterized by at least three hierarchically ordered scales, $mv^2 \ll mv \ll m$, where m is the mass of the heavy quark and v its velocity. Therefore, it is only under the condition $\Lambda_{\text{QCD}} \ll mv^2$ that the use of local condensates and, hence, the Voloshin–Leutwyler formula (1) can be justified. In the other regimes where heavy quarkonium states can sit, the non-perturbative dynamics will be encoded into more extended objects: non-local condensates and Wilson loop operators. This is the case for higher quarkonium levels ($n > 1$).

In the following we will consider non-perturbative effects in heavy quarkonium systems in different kinematic regimes, in an effective field theory context. This approach has not only the considerable practical advantage of disentangling the different dynamical scales of the system, but also the conceptually relevant feature of disentangling perturbative effects from non-perturbative ones. This allows us to fully exploit the predictive power of QCD.

2 Local and non-local condensates: $\Lambda_{\text{QCD}} \ll mv$

For the lower lying quarkonium states, it can be expected that the inverse of the typical size of the system is larger than Λ_{QCD} . If this condition is fulfilled, both scales m and mv can be integrated out perturbatively from QCD, leading to an effective field theory where only the degrees of freedom of order Λ_{QCD} or mv^2 remain dynamical. This effective field theory is known as potential NRQCD, pNRQCD^{5,6}.

The infrared sensitivity of the quark–antiquark static potential at three loops⁷ signals that it may become sensitive to non-perturbative effects if the next relevant scale after mv is Λ_{QCD} . Indeed, in the situation $mv \gg \Lambda_{\text{QCD}} \gg mv^2$, the leading non-perturbative contribution (in α_s and in the multipole expansion) to the static potential reads^{6,8}

$$V_0(r)^{\text{non-pert}} = -i \frac{g^2}{N_c} T_F \frac{r^2}{3} \int_0^\infty dt e^{-itC_A \alpha_s/(2r)} \langle \mathbf{E}^a(t) \phi(t, 0)_{ab}^{\text{adj}} \mathbf{E}^b(0) \rangle(\mu), \quad (3)$$

where r is the quark–antiquark distance. This term explicitly cancels, up to the considered order, the dependence of the perturbative static potential on the infrared scale μ . It is interesting to note that the leading contribution in the

$\Lambda_{\text{QCD}}/mv^2$ expansion of $V^{\text{non-pert}}$ (obtained by putting the exponential equal to 1) cancels the order $\Lambda_{\text{QCD}}^3 r^2$ renormalon that affects the static potential (the leading-order renormalon, of order Λ_{QCD} , cancels against the pole mass). Therefore, also in renormalon language, the above operator is the relevant non-perturbative contribution to the static potential in the considered kinematic situation.

If $\Lambda_{\text{QCD}} \lesssim mv^2$ the static potential is purely perturbative and its explicit dependence on the infrared scale μ is reabsorbed in a physical observable by non-potential contributions. In the specific case of the quarkonium energy levels up to order $\alpha_s^5 \ln \mu$, these contributions are⁹

$$\delta E_{n,l,j} = -i \frac{g^2}{3N_c} T_F \int_0^\infty dt \langle n, l | \mathbf{r} e^{it(E_n - H_o)} \mathbf{r} | n, l \rangle \langle \mathbf{E}^a(t) \phi(t, 0)_{ab}^{\text{adj}} \mathbf{E}^b(0) \rangle(\mu), \quad (4)$$

$$E_n \equiv -m \frac{C_F^2 \alpha_s^2}{4n^2}, \quad H_o \equiv \frac{\mathbf{p}^2}{m} + \frac{1}{2N_c} \frac{\alpha_s}{r},$$

where $|n, l\rangle$ are the Coulomb wave functions. It is worth while to notice that, if $\Lambda_{\text{QCD}} \ll mv^2$, the scale mv^2 can be integrated out perturbatively from the above formula and the non-perturbative contributions reduce to the Voloshin-Leutwyler formula (1), as can easily be seen by recognizing that:

$$m \frac{\epsilon_n n^6}{(m C_F \alpha_s)^4} = \frac{1}{3N_c} T_F \left\langle n, l \left| \mathbf{r} \frac{1}{E_n - H_o} \mathbf{r} \right| n, l \right\rangle.$$

The above outline leads to the following conclusion. For quarkonium of a typical size smaller than $1/\Lambda_{\text{QCD}}$, the most relevant operator of the non-perturbative dynamics is the bilocal gluon condensate $\langle \mathbf{E}^a(t) \phi(t, 0)_{ab}^{\text{adj}} \mathbf{E}^b(0) \rangle$, which belongs to the class of non-local gluon condensates considered in the introduction. In the following section we will discuss our present knowledge of it.

2.1 The non-local condensate $\langle F_{\mu\nu}^a(x) \phi(x, 0)_{ab}^{\text{adj}} F_{\lambda\rho}^b(0) \rangle$

The correlator $\langle F_{\mu\nu}^a(x) \phi(x, 0)_{ab}^{\text{adj}} F_{\lambda\rho}^b(0) \rangle$ is perturbatively known at the next-to-leading order in α_s ¹⁰. However, here we are interested in its non-perturbative behaviour. Different parametrizations have been proposed^{12,13,14,15,16}. Because of its Lorentz structure, the correlator is in general described by two form factors. A convenient choice of these consists in the chromoelectric and chromomagnetic correlators:

$$\langle \mathbf{E}^a(x) \phi(x, 0)_{ab}^{\text{adj}} \mathbf{E}^b(0) \rangle, \quad \langle \mathbf{B}^a(x) \phi(x, 0)_{ab}^{\text{adj}} \mathbf{B}^b(0) \rangle.$$

The strength of the correlators is of the order of the gluon condensate. In the long range ($x^2 \rightarrow \infty$) they fall off exponentially (in the Euclidean space) with some typical correlation lengths. In the following we will concentrate on these correlation lengths.

The lattice calculation¹⁵, using cooling techniques, obtains the same correlation length (T_g) for both form factors, and this is

$$T_g = 0.34 \pm 0.02 \pm 0.03 \text{ fm} \quad (4 \text{ flavours, } am = 0.01), \quad (5)$$

$$T_g = 0.22 \pm 0.01 \pm 0.02 \text{ fm} \quad (\text{quenched}). \quad (6)$$

The less accurate, but traditional (quenched) lattice calculation done in¹⁶ obtains two different correlation lengths for the chromoelectric (T_g^E) and the chromomagnetic correlators (T_g^B):

$$T_g^E \neq T_g^B \simeq 0.1\text{--}0.2 \text{ fm} \quad (\text{quenched}). \quad (7)$$

Finally, a recent sum-rule estimation¹⁴ obtains $T_g^E < T_g^B$. The sum rule turns out not to be stable for the chromoelectric correlator, while for the chromomagnetic correlation length it gives

$$T_g^B = 0.13_{-0.02}^{+0.05} \text{ fm} \quad (3 \text{ flavours}), \quad (8)$$

$$T_g^B = 0.11_{-0.02}^{+0.04} \text{ fm} \quad (\text{quenched}). \quad (9)$$

The correlation lengths T_g^E and T_g^B have a precise physical interpretation. Their inverses correspond to the masses of the lowest-lying vector and pseudovector static quark–gluon hybrids, respectively. This can be explicitly seen in the short-range limit, $\mathbf{x} \rightarrow 0$, where the hybrids (in this case also called gluelumps) operators can be explicitly constructed^{6,17}. The suitable effective field theory is pNRQCD in the static limit⁶. Gluelump operators are of the type $\text{Tr}\{OH\}$, where $O = O^a T^a$ corresponds to a quark–antiquark state in the adjoint representation (the octet) and $H = H^a T^a$ is a gluonic operator. By matching the QCD static hybrid operators into pNRQCD, we get the static energies (also called potentials) of the gluelumps. At leading order in the multipole expansion, they read

$$V_H(r) = V_o(r) + \frac{1}{T_g^H}, \quad (10)$$

$$\langle H^a(t)\phi(t,0)_{ab}^{\text{adj}} H^b(0) \rangle^{\text{non-pert.}} \simeq h e^{-it/T_g^H} + \dots$$

Since hybrids are classified in QCD according to the representations of $D_{\infty,h}$, while in pNRQCD, where we have integrated out the length r , their classification is done according to the representations of $O(3) \times C$, the static hybrid

short-range spectrum is expected to be more degenerate than the long-range one^{18,17}. The lattice measure of the hybrid potentials done in¹⁹ confirms this feature. In⁶ it has been shown that the quantum numbers attribution of pNRQCD to the short-range operators, and the expected $O(3) \times C$ symmetry of the effective field theory match the lattice measurements. By using only \mathbf{E} and \mathbf{B} fields and keeping only the lowest-dimensional representation we may identify the operator H for the short-range hybrids called $\Sigma_g^{+'}$ (and Π_g) with $\mathbf{r} \cdot \mathbf{E}$ (and $\mathbf{r} \times \mathbf{E}$) and the operator H for the short-range hybrids called Σ_u^- (and Π_u) with $\mathbf{r} \cdot \mathbf{B}$ (and $\mathbf{r} \times \mathbf{B}$). Hence, the corresponding static energies for small r are

$$V_{\Sigma_g^{+'}, \Pi_g}(r) = V_o(r) + \frac{1}{T_g^E}, \quad V_{\Sigma_u^-, \Pi_u}(r) = V_o(r) + \frac{1}{T_g^B}.$$

The lattice measure of¹⁹ shows that, in the short range, $V_{\Sigma_g^{+'}, \Pi_g}(r) > V_{\Sigma_u^-, \Pi_u}(r)$. This supports the sum-rule prediction¹⁴ that the pseudovector hybrid lies lower than the vector one, i.e. $T_g^E < T_g^B$.

3 Wilson loop operators: $\Lambda_{\text{QCD}} \sim mv$

For higher quarkonium levels, Λ_{QCD} is expected to be comparable with mv . We cannot match into pNRQCD perturbatively, since the scale associated to the quarkonium size r is already non-perturbative. The relevant non-perturbative dynamics is therefore contained in more extended objects than (local or non-local) gluon condensates: Wilson loops and field insertions on these.

In particular, disregarding effects due to scales lower than Λ_{QCD} , the static potential is given by²⁰

$$V_0(r) = \lim_{T \rightarrow \infty} \frac{i}{T} \ln \langle W_\square \rangle, \quad (11)$$

where W_\square is the static Wilson loop of size $r \times T$ and $\langle \rangle$ means an average over the gauge fields. Lattice studies tell us that at distances $r \simeq 1/\Lambda_{\text{QCD}}$ the potential is no longer Coulombic but rises linearly ($V_0(r) \simeq \sigma r$). Higher-order corrections in the $1/m$ expansion have been calculated over the years²¹ and are given by field strength insertions on the Wilson loop. For instance the next-to-leading potential in the $1/m$ expansion is²²

$$\frac{V_1}{m} = \frac{1}{m} \lim_{T \rightarrow \infty} \left(-\frac{g^2}{4T} \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' |t - t'| \left[\langle \langle \mathbf{E}(t) \cdot \mathbf{E}(t') \rangle \rangle_\square \right. \right. \quad (12)$$

$$\left. - \langle \langle \mathbf{E}(t) \rangle \rangle_\square \cdot \langle \langle \mathbf{E}(t') \rangle \rangle_\square \right] \Bigg), \quad (13)$$

where $\langle\langle \rangle\rangle_\square$ means a normalized gauge average in the presence of the static Wilson loop.

Wilson-loop operators of the above type may be interpreted as a superposition of states, describing gluonic excitations between static sources²². They have been so far evaluated only inside QCD vacuum models or by lattice simulations (for some reviews, see²³ and also²⁴).

3.1 The stochastic expansion

We can now ask if some relation can be established between the Wilson loop (and field strength insertions on it) and the non-local gluon condensates discussed above. In fact we can express the Wilson loop as a formal expansion in terms of gluonic correlation functions by means of the so-called stochastic expansion^{11,12}

$$\ln\langle W_\square \rangle = \sum_{n=0}^{\infty} \frac{(ig)^n}{n!} \int_{S(\square)} dS_{\mu_1\nu_1}(u_1) \cdots dS_{\mu_n\nu_n}(u_n) \langle \phi(0, u_1) \times F^{\mu_1\nu_1}(u_1) \phi(u_1, 0) \cdots \phi(0, u_n) F^{\mu_n\nu_n}(u_n) \phi(u_n, 0) \rangle_{\text{cum}}, \quad (14)$$

where $S(\square)$ denotes a surface whose contour the rectangular Wilson loop. The cumulants $\langle \rangle_{\text{cum}}$ are defined as

$$\begin{aligned} \langle \phi(0, u_1) F(u_1) \phi(u_1, 0) \rangle_{\text{cum}} &= \langle \phi(0, u_1) F(u_1) \phi(u_1, 0) \rangle = 0, \\ \langle \phi(0, u_1) F(u_1) \phi(u_1, u_2) F(u_2) \phi(u_2, 0) \rangle_{\text{cum}} &= \\ &\langle \phi(0, u_1) F(u_1) \phi(u_1, u_2) F(u_2) \phi(u_2, 0) \rangle \\ &- \langle \phi(0, u_1) F(u_1) \phi(u_1, 0) \rangle \langle \phi(0, u_2) F(u_2) \phi(u_2, 0) \rangle \dots = \\ &\langle F(u_1) \phi(u_1, u_2)^{\text{adj}} F(u_2) \rangle, \\ &\dots \quad \dots \end{aligned}$$

It is important to realize that the expansion of Eq. (14) is substantially different with respect to the expansions in $1/m$ and r , which led to the construction of the low-energy effective field theories discussed above. Those expansions were justified by the dynamics of the system under study, and each term of it is, indeed, suppressed by powers of the highest dynamical scale left divided by the scale (which is larger) that has been integrated out. Instead, from a power counting point of view, each term of the expansion (14) is of the same size (for instance each term of the series is in general expected to contribute to the string tension σ). As soon as we truncate the series, e.g. up to the bilocal cumulant (the first non-vanishing one), we introduce an uncontrolled approximation and define a model. This model is known as the model of the

stochastic vacuum¹². Indeed, this model turns out to be quite successful in the study of processes that involve quark systems that may be described by almost static Wilson loops (for some reviews, see²⁵). Let us only mention that the model predicts, in agreement with the lattice data, a long-range linear static potential with slope $\sigma \sim T_g^{E2} G_2$. It would be highly desirable to have a field theoretical justification of the expansion (14); however, such a justification is missing up to now (for a recent investigation on higher cumulants, see²⁶).

4 Conclusions

In the previous sections we have shown how the non-perturbative QCD vacuum enters the dynamics of heavy quarkonium. As much as the non-perturbative scale Λ_{QCD} is bigger than the dynamical scales of the non-relativistic system (m , mv and mv^2), as extended the relevant non-perturbative operators are. In the situation $\Lambda_{\text{QCD}} \ll mv^2$ these operators reduce to local condensates, i.e. some numbers. This is the most favourable situation for a theoretical investigation. The bottomonium and charmonium ground states have been investigated in this framework (for a recent review, see²⁷). In the limiting case of $t\bar{t}$ threshold production the non-perturbative corrections can be neglected and the investigation is completely accessible to perturbative QCD²⁸. In the situation $mv^2 \lesssim \Lambda_{\text{QCD}} \ll mv$ the non-perturbative physics is encoded into non-local condensates. The dominant one is the bilocal gluon condensate. As we have discussed above it essentially depends on its strength, i.e. the gluon condensate, and on some correlation lengths, which can be related to the masses of the lowest-lying (static) quark-gluon hybrid resonances. Although this situation seems quite interesting, it has been poorly investigated in heavy quarkonium phenomenology, and mainly in the framework of models^{8,4}. The main reason is, in our opinion, that, while the quarkonium ground state seems to be accessible by a purely perturbative treatment (plus local condensates) and the potential describing higher excited quarkonium states seems entirely dominated by non-perturbative effects, it is, a priori, not clear to which states the situation $mv^2 \lesssim \Lambda_{\text{QCD}} \ll mv$ is applicable. We will come back to this point at the end of this section. Finally, in the situation $\Lambda_{\text{QCD}} \sim mv$ the relevant non-perturbative objects are Wilson-loop operators. A lattice study of them and the subsequent quarkonium spectroscopy have been done in²⁹.

The outlined study is rigorous and allows a systematic disentanglement of the high from the low energy scales of the heavy quarkonium system under study. In this specific sense also perturbative and non-perturbative effects turn out to be disentangled.

We conclude by mentioning a somehow delicate point, connected with the

scales of heavy quarkonium systems: the difficulty to state *a priori* in which kinematic situation a particular quarkonium state is. This is mainly due to the fact that the scales mv and mv^2 are not so well defined, nor so widely separated that different kinematic situations cannot overlap in a preliminary analysis (for a related discussion on the energy scales also in quarkonium physics, see³⁰). Therefore, there are situations in which scales can only be fixed *a posteriori*, i.e. by assuming a particular situation and by checking that the final result is consistent with it and with the experimental data.

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